Visualization in the Teaching and Learning of Mathematics

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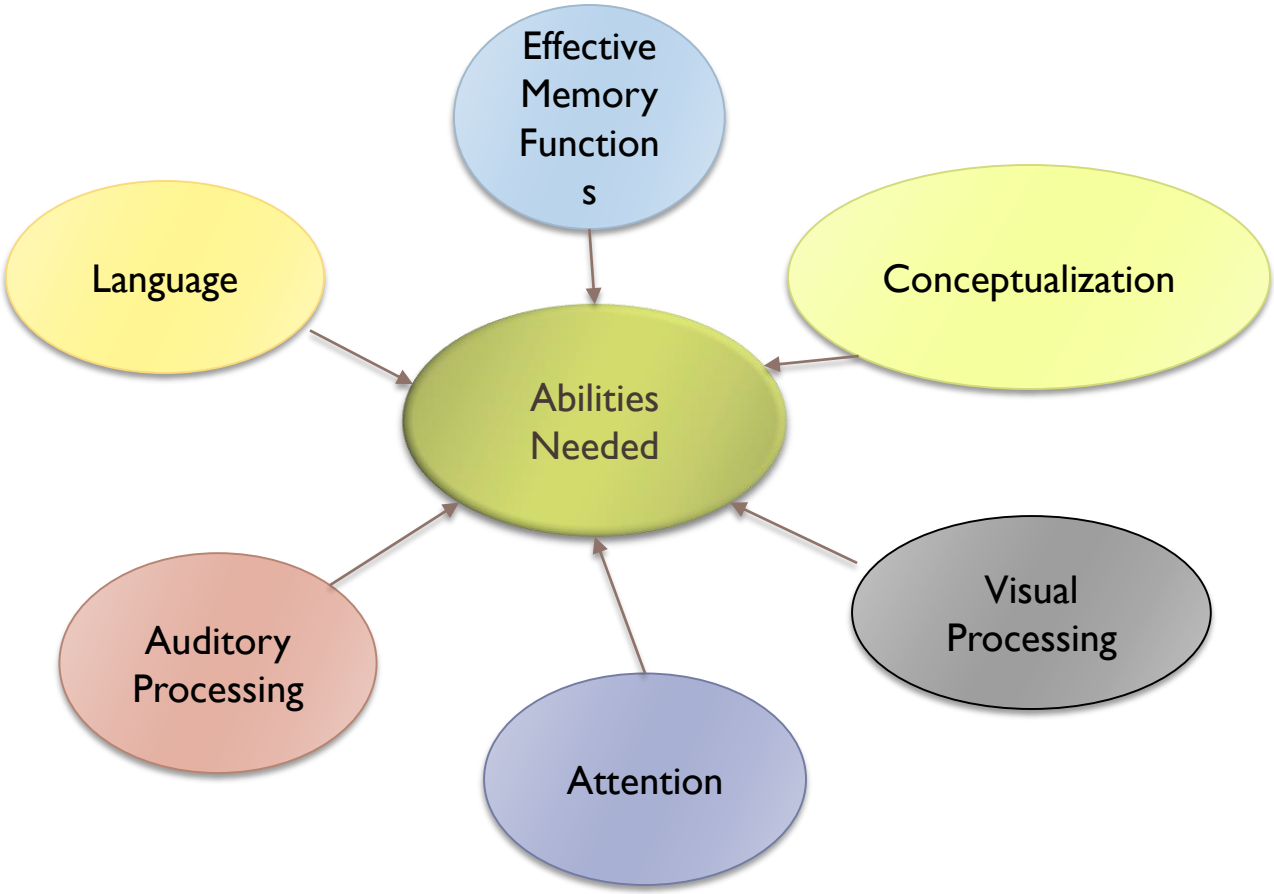
What is Math?

- The science of quantity
- The study of patterns
- Any kind of regularity that can be recognized by the mind
- A universal tool for problem solving and discovery
- Thinking with numbers, imagery and language

What are the Abilities Needed?



Solve the following

You have 30 seconds:

$$\begin{array}{r} 362 \\ \times\ 12 \\ \hline \\ \hline \end{array}$$

Processing Model

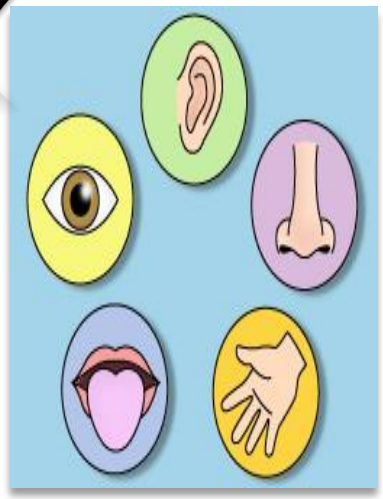
Simplification of a complex task

Input

Vision
+
Hearing

Processing & integration of:

Sensory
Visual
Auditory
Information



Memory:

Short / working memory

Long Term Memory

Auditory-visual-tactile memory

Executive Fx:

Cognition
(Non-verbal
Verbal)

Reasoning
(Divergent
Convergent)

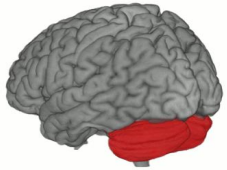
Problem solving

Planning

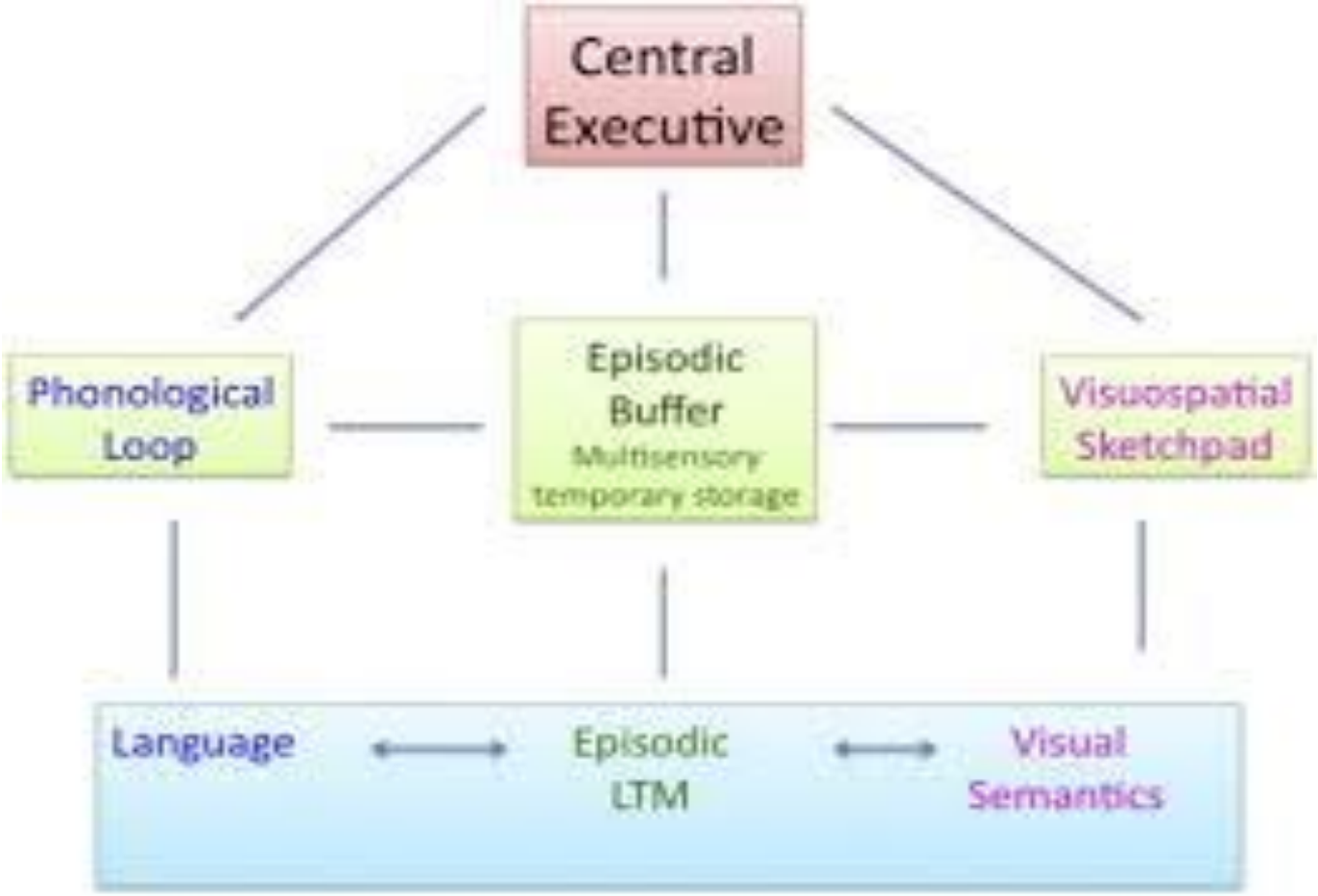
Output

Motor
+
Vision

Attention + Social / Emotional Development / Personality



Working Memory



Working memory:

Holds and pays attention to information that is sensed, or that is triggered from old memories.	Holds and switches between bits of information when they are needed. Ignores information that isn't.	Holds options in mind while working out what needs to be done. Then holds details to plan then start activity.	Holds goals in mind to check if the plan is working.

Processing speed

What is it to Visualize?

- **verb**

1.form a mental image of; imagine.

- "it is not easy to visualize the future"
- *synonyms:* envisage, envision, conjure up, conjure up an image/picture of, picture in the mind's eye, picture, call to mind, see, imagine, evoke, fancy, dream about, dream up, fantasize about, conceptualize, conceive of, think about, contemplate

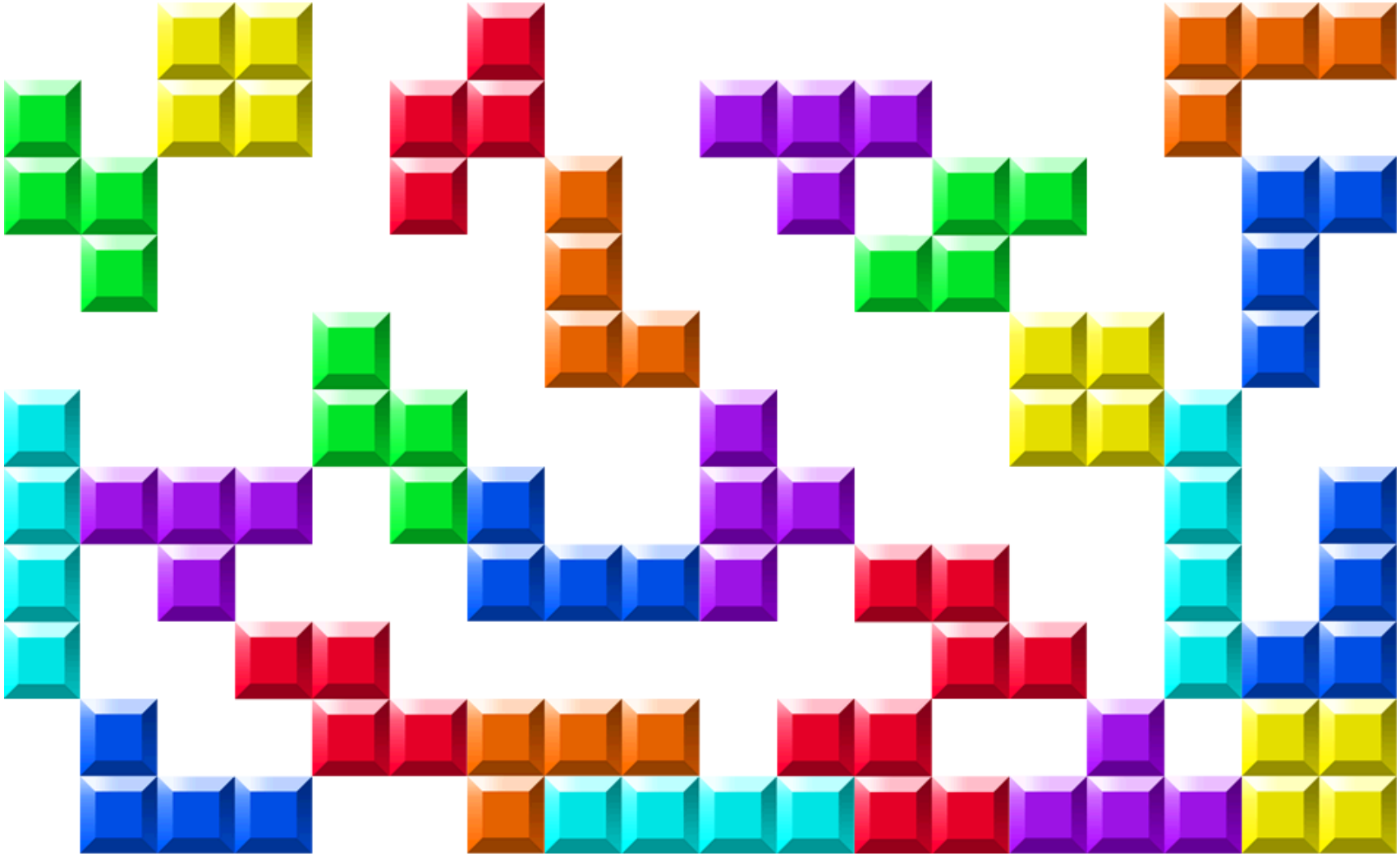
The Visuospatial Sketchpad, what is it?

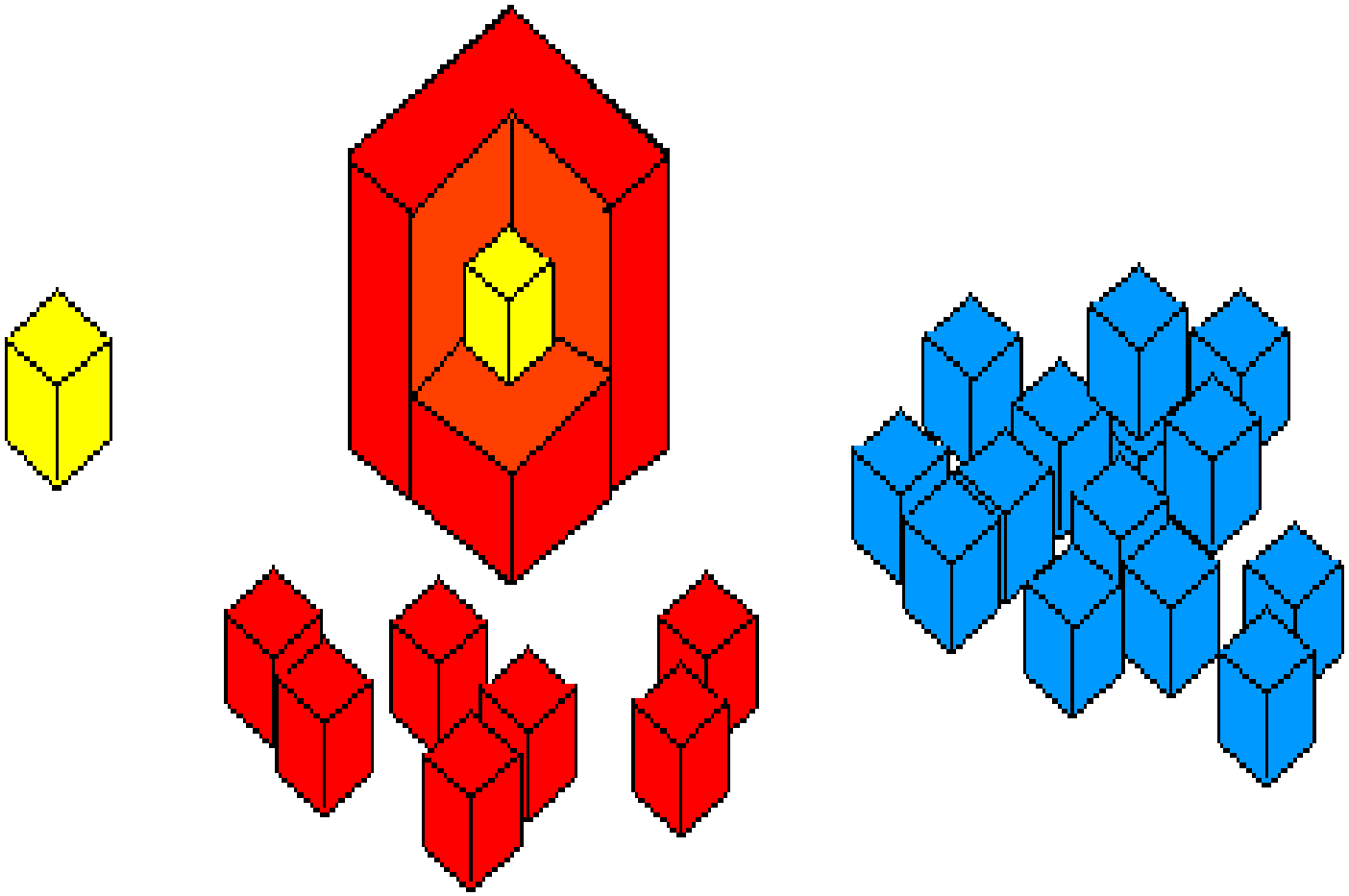
- The Visuo-spatial sketchpad (VSS) is an important element in the function of working memory, as it is responsible for storing and processing information in visual or spatial form, as well as the location or speed of objects in space.

- It has been proposed that the VSS can be further subdivided into 2 separate visual and spatial components:
- **the visual cache** – stores information on form and colour, and
- **the inner scribe** – focuses on spatial and movement information. The inner scribe also rehearses information in the visual cache and transfers information to the central executive.

Function

- The Visuo-spatial sketchpad is used in ways such as: temporarily storing information on how things look, for example, shapes and colours, and is also what allows us to manipulate images in our mind. This can be seen in a game of Tetris, as one rotates a shape to see how it might fit or appear from a different angle.





Imagery: the Sensory-Cognitive Connection for Math

- For individuals who "get math", the language of numbers turns into imagery and they "see" mathematical relationships.
- Mathematics is cognitive processing, thinking, that requires the dual coding of imagery and language. Imagery is fundamental to the process of thinking with numbers.
- Dual coding in math requires two aspects of imagery: symbol/numeral imagery (parts/details) and concept imagery (whole/gestalt). Perhaps the two imagery systems reside in slightly different areas of the brain.

Why Visualize?

- The main intent of information visualization is to represent an abstract information space in a dynamic way, so as to facilitate human interaction for exploration and understanding.

- Information visualization makes use of the principles in Gestalt Theory regarding the human visual capacity as a powerful pattern-finding engine, to provide a powerful means of making sense of the abundance of available data.

- Visual learning is about acquiring and communicating information. We easily understand that information often comes to us through words and numbers. We readily acknowledge that students must be taught to read and understand and use these ways of conveying data.

- But we don't often pay as much attention to the ways information can be conveyed visually: through illustrations, photos, diagrams, graphs, symbols, icons, and other visual representations.

Periodic Table

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H Hydrogen 1.008																	2 He Helium 4.00260
2	3 Li Lithium 6.940	4 Be Beryllium 9.01218											5 B Boron 10.810	6 C Carbon 12.011	7 N Nitrogen 14.007	8 O Oxygen 15.999	9 F Fluorine 18.99840	10 Ne Neon 20.1797
3	11 Na Sodium 22.98977	12 Mg Magnesium 24.305											13 Al Aluminum 26.98154	14 Si Silicon 28.085	15 P Phosphorus 30.97376	16 S Sulfur 32.060	17 Cl Chlorine 35.450	18 Ar Argon 39.948
4	19 K Potassium 39.0983	20 Ca Calcium 40.078	21 Sc Scandium 44.95591	22 Ti Titanium 47.867	23 V Vanadium 50.9415	24 Cr Chromium 51.9961	25 Mn Manganese 54.93804	26 Fe Iron 55.845	27 Co Cobalt 58.93319	28 Ni Nickel 58.6934	29 Cu Copper 63.546	30 Zn Zinc 65.38	31 Ga Gallium 69.723	32 Ge Germanium 72.630	33 As Arsenic 74.92159	34 Se Selenium 78.971	35 Br Bromine 79.904	36 Kr Krypton 83.798
5	37 Rb Rubidium 85.4678	38 Sr Strontium 87.62	39 Y Yttrium 88.90584	40 Zr Zirconium 91.224	41 Nb Niobium 92.90637	42 Mo Molybdenum 95.95	43 Tc Technetium [97.90721]	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.90550	46 Pd Palladium 106.42	47 Ag Silver 107.8682	48 Cd Cadmium 112.414	49 In Indium 114.818	50 Sn Tin 118.710	51 Sb Antimony 121.760	52 Te Tellurium 127.60	53 I Iodine 126.90447	54 Xe Xenon 131.293
6	55 Cs Cesium 132.90545	56 Ba Barium 137.327	71 Lu Lutetium 174.9668	72 Hf Hafnium 178.49	73 Ta Tantalum 180.94788	74 W Tungsten 183.84	75 Re Rhenium 186.207	76 Os Osmium 190.23	77 Ir Iridium 192.217	78 Pt Platinum 195.084	79 Au Gold 196.96657	80 Hg Mercury 200.592	81 Tl Thallium 204.380	82 Pb Lead 207.2	83 Bi Bismuth 208.98040	84 Po Polonium [209]	85 At Astatine [210]	86 Rn Radon [222]
7	87 Fr Francium [223]	88 Ra Radium [226]	103 Lr Lawrencium [262]	104 Rf Rutherfordium [267]	105 Db Dubnium [268]	106 Sg Seaborgium [271]	107 Bh Bohrium [274]	108 Hs Hassium [269]	109 Mt Meitnerium [276]	110 Ds Darmstadtium [281]	111 Rg Roentgenium [281]	112 Cn Copernicium [285]	113 Nh Nihonium [286]	114 Fl Flerovium [289]	115 Mc Moscovium [288]	116 Lv Livermorium [293]	117 Ts Tennessine [294]	118 Og Oganesson [294]

57 La Lanthanum 138.90547	58 Ce Cerium 140.116	59 Pr Praseodymium 140.90766	60 Nd Neodymium 144.242	61 Pm Promethium [144.91276]	62 Sm Samarium 150.36	63 Eu Europium 151.964	64 Gd Gadolinium 157.25	65 Tb Terbium 158.92535	66 Dy Dysprosium 162.500	67 Ho Holmium 164.93033	68 Er Erbium 167.259	69 Tm Thulium 168.93422	70 Yb Ytterbium 173.04
89 Ac Actinium [227]	90 Th Thorium 232.0377	91 Pa Protactinium 231.03588	92 U Uranium 238.02891	93 Np Neptunium [237]	94 Pu Plutonium [244]	95 Am Americium [243]	96 Cm Curium [247]	97 Bk Berkelium [247]	98 Cf Californium [251]	99 Es Einsteinium [252]	100 Fm Fermium [257]	101 Md Mendelevium [258]	102 No Nobelium [259]

- Mendeleev's periodic table of elements, which encodes several types of data in a small table format, is probably one of the most famous examples of visualization used in educational contexts
- This visualization is informative, efficient and can be considered one of the earlier beautiful visualizations of complex chemistry data (Steele & Iliinsky, 2010).
- Mendeleev's periodic table visualization is known by millions of students all over the world and is a perfect example how visualization can be effectively used to support understanding of subject matter

Visual Literacy

- The skill that allows us to interpret this language is visual literacy. Students need to be taught to be visually literate, exactly as we teach them to be verbally literate.

- They must learn how to read, interpret, and produce visual data. Visual literacy is about learning to read our visual world. (Alfano, 2005)

Visual learning involves a specific set of skills. These include:

1. Observation,
2. Recognition,
3. Perception,
4. Interpretation, and
5. Self-expression or communication.

- **Observation** is to really see something and to examine its attributes. Observation answers the questions "What is it?" and "What makes it that?"

- **Recognition** includes visual recall. It allows a student to say "That symbol means 'stop'" or "That's a triangle because it has three corners and three sides."

- **Interpretation** leads to comprehension and understanding. Interpretation involves the questions "What does that model tell me?" or "How does it work?"

- **Perception** deals with analysis and conjecture. It enables a student to answer the question "What will happen next?"

- **Self-expression** relates to using visual techniques to convey ideas to others: sketching, making images, drawing diagrams, and creating charts. "How can I get this idea across?" is the question that this skill answers.

What Should Teachers Do?

- Teachers can display materials that help to convey ideas visually—charts, graphs, timelines, and diagrams. They can encourage students to draw images to explain or solve problems. They can ask students to work together to model new concepts. Sketching should be part of a daily math note-taking process. Drawing out a concept and visualizing how it works leads to comprehension. (Armstrong, 1994)

4 Strategies to Help Kids Understand Math Using Visualization

1. **Be Strategic About Using Manipulatives:**

- Manipulatives really help to create a visual representation of a math concept you're teaching. The next step is to help students store those visual images in their brains. For example, when you're helping kids learn about where numbers fit on a number line, you can use Unifix cubes to build the number line.

2. Make Your Number Line Life-Size:

- Sure, there are number lines in your textbook. But how about one that your students can walk on! With a number line, students see and learn the sequence and pattern of numbers in our number system. They see what number comes after 5 and what number comes before 5. They see what the number 15 has in common with the number 25. Building a number line on the floor gives students the opportunity to interact with the numbers in a tactile way, which helps them visualize the numbers and their values, even when the number line is gone.

3. Give Students the Answer

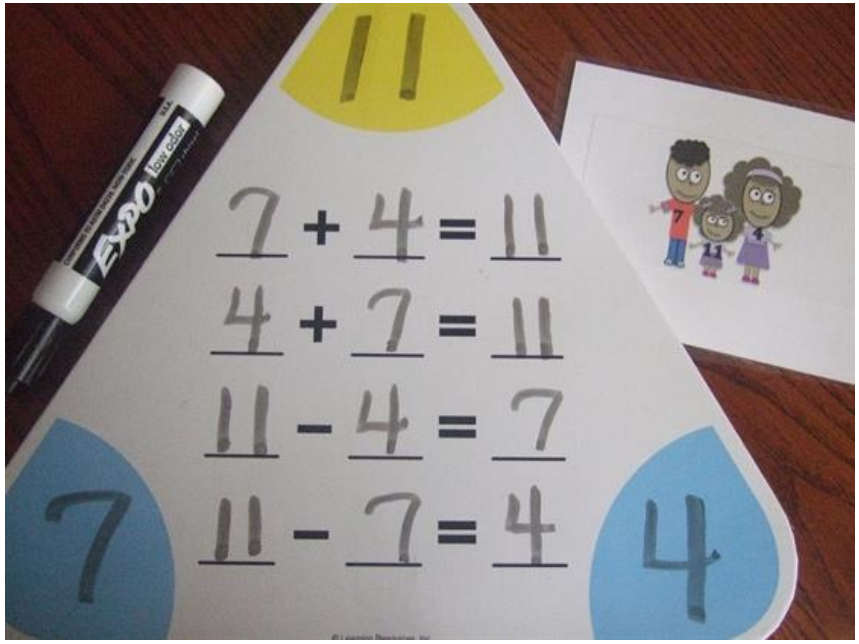
- It might sound surprising, but actually *giving* kids the answers as you help them learn the facts is a strategy that works well for some children. One way to help fact families "stick" is to teach the family as a unit, including all of the answers. Here's where it gets different: You can also ask the children to rehearse the fact with the answer visible. Asking the child to create a visual image of a math problem (complete with answer) and then re-create that problem visually, even with your example gone, can help facilitate fact recall.

4. Introduce Fact Family Cards

- A twist on this strategy involves fact family cards. Fact family cards have the facts of the family but not a sign. This way, the cards can be used for both adding and subtracting. Cover the answer for the problem you wish the child to solve with your thumb and then ask them to solve either the addition or subtraction version of the fact family. In the example below, if you cover the 10, then you will be asking the child to solve the addition problem of 3 + 7 or the problem of 7 + 3.

- Visual learning strategies can make a profound difference in a student's depth of understanding about mathematics. It is a powerful teaching tool for those who are natural visual/spatial learners, for those who are English language learners, and for all students. In fact, by using visual learning strategies in the teaching of mathematics, we can—and we should—increase the learning potential of all students and develop their ability to communicate mathematical concepts in an increasingly visual world.

Fact Family Cards



- THANK YOU!